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A STATISTICAL BASIS FOR CONVECTIVE STORM PROBABILITY FORECASTS

by

W.S. HARLEY

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ABSTRACT

A convective storm diagnostic and prediction technique is developed using instability indices highly correlated with convective storm occurrence. Several forms of an instability index based on the equivalent potential temperature are compared statistically with other indices in common use. Polynomial regression curves of best fit are employed to determine probabilities of convective storm occurrence from instability index values. These polynomials may be used as a basis for convective storm probability forecasts.

FONDEMENTS STATISTIQUES DE LA PRÉVISION DE PROBABILITÉ D'ORAGES CONVECTIFS

par

W.S. Harley

RÉSUMÉ

Il s'agit ici d'établir une technique de diagnostic et de prévision des orages convectifs au moyen d'indices d'instabilité qui correspondent nettement à l'apparition de ces orages. On fait une comparaison statistique entre plusieurs types d'indices d'instabilité fondés sur la température potentielle équivalente, et d'autres indices couramment utilisés. Pour déterminer les probabilités d'un orage convectif à partir des valeurs d'indices d'instabilité, on utilise les courbes polynomiales de régression qui conviennent le mieux; ces polynômes peuvent servir de fondement à la prévision de probabilité d'orages convectifs.

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1. Introduction

A project to develop a method of convective storm prediction was commenced in 1969 as a result of a request from the Winnipeg Weather Central for an evaluation of a Severe Weather Project carried out by them in the summer of 1968. This latter project was based exclusively on the severe storms forecast system developed by the National Severe Storms Forecast Center, Kansas City, to predict the severe storms and tornadoes that occur in certain parts of the United States. The technique appears applicable in Canada in unmodified form only on the Prairies where such storms are well known. Consequently, it was decided to devise a method of storm forecasting suited to conditions in areas of Canada other than the Prairies. In pursuit of this aim, several forms of an index based on the equivalent potential temperature were developed and tested against other indices in common use to determine their suitability as indicators of convective storms of all intensities.

The work was started under the authority of the Scientific Development and Evaluation Unit of the Training Branch, Canadian Meteorological Service and continued in the Meteorological Services Research Branch after being incorporated into it in 1970. Ten months of data for the study were obtained from 74 radiosonde stations in North American between 35 and 65°N. No data for levels above 400 mb were used for reasons of economy.

This paper describes work carried out and results obtained between 1969 and 1974.

2. Development and Use of an Instability Energy Index

It has been shown that instability showers and thunderstorms may form in ascending currents of moist air within an unstable $\delta > \delta_d$, or conditionally unstable $\delta > \delta_m$, atmosphere, where δ, δ_m and δ_d indicate the actual, moist and dry adiabatic lapse rates of temperature, respectively. In particular, thunderstorms will occur whenever there is a steepening of the lapse rate of temperature in very moist layers of sufficient depth; see Byers (1959), and Petterssen (1958).

Byers (1959), listed the processes that may cause steepening of the lapse rate, e.g., general upward motion may steepen the lapse rate with great rapidity in a layer having pronounced moisture stratification. Such a layer is said to be convectively or potentially unstable. Rossby (1932) has shown that a layer will be convectively unstable (or have potential instability) if the equivalent potential temperature decreases with height throughout the layer, i.e.,

$$\frac{d\theta_E}{dz} < 0 \qquad (1)$$

If several processes that cause steepening of the lapse rate, such as warm air advection at low levels, cold air advection aloft and lifting, act simultaneously in one area, very intense thunderstorms may be produced (Newton, 1967; Palmén and Newton, 1969). The formation of a severe hail- and tornado-producing thunderstorm also requires some combination of potential and conditional instability throughout a deep layer of the troposphere (Palmén and Newton, 1969). Either type of instability may occur without the other, in which case severe convective storms are unlikely to accompany release of the instability. An instability index formed from the sum of the conditional and potential instabilities may be a useful tool for predicting these storms. Such an index is derived below.

The equivalent and saturated equivalent potential temperatures (θ_E and θ_{sE}) may be used to derive this index, since both are readily computed from the following equations (Iribarne and Godson, 1973):

$$\theta_E = \theta + \frac{B \cdot p_o}{p} e_w(T_d), \tag{2}$$

$$\theta_{sE} = \theta + \frac{B \cdot p_o}{p} e_w(T), \tag{3}$$

where θ is the potential temperature of the air; $B = 0.62197 \times L_v/C_p p_o = 1.555°C\ mb^{-1}$; L_v is the latent heat of water vapour; C_p is the specific heat of air at constant pressure; p_o is 1000 mb; $e_w(T_d)$, $e_w(T)$ are the water vapour pressures at dew point T_d and temperature T for air at pressure level p. Both θ_E and θ_{sE} are ideally suited for computer analysis of stability characteristics.

Latent instability is the most important type of conditional instability and it may be computed from a modification of the saturated equivalent potential temperature form of the conditional instability equation; see Equation (101), Iribarne and Godson (1973). The numerator of the finite-difference form of this equation is replaced by the expression: $\theta_{sE}(p') - \theta_E$, thus:

$$\sigma_{sE} = -\frac{1}{\overline{\theta_{sE}}} \cdot \frac{\theta_{sE}(p') - \theta_E}{p' - p} < 0, \tag{4}$$

where $\theta_{sE}(p') - \theta_E$ is a measure of the latent instability in a layer comparable to but not identical with the Showalter (1953) Index; see Godson (1958) and Iribarne and Godson (1973, p. 185); σ_{sE} is the latent instability parameter; $\overline{\theta_{sE}}$ is the mean value of $\theta_{sE}(p')$ and θ_E. Latent instability will be present if $\theta_E > \theta_{sE}(p')$ for some $p' < p$. Equation (4) may be used to determine the degree of latent instability in an air column. Its value will be called the Latent Instability index.

The potential instability in a layer may be computed from the finite-difference mode of the equivalent potential temperature form of the potential instability equation; see Equation (101), Iribarne and Godson (1973), thus:

$$\sigma_E = -\frac{1}{\overline{\theta_E}} \cdot \frac{\theta_E(p') - \theta_E}{p' - p} < 0, \tag{5}$$

where $\bar{\theta}_E$ is the mean value of $\theta_E(p')$, and θ_E and the other symbols have the same meaning as before. Potential instability will be present if $\theta_E > \theta_E(p')$ for some $p' < p$. The value given by (5) will be called the Potential Instability index.

The potential instability, as measured by (5), is related to the release of the total energy (E_T) in a thunderstorm via θ_E which is directly porportional to it, as follows (Darkow, 1967):

$$\theta_E = E_T/C_p = T + 9.8Z + 2.5r_w \qquad (6)$$

where Z is the height (m) and r_w is the mixing ratio of the air (g kg^{-1}) at temperature T (K).

In (5), θ_E represents the equivalent potential temperature of the warm, moist low-level air entering the updraft of a thunderstorm, while $\theta_E(p')$ represents that of the cold, dry air in the downdraft (Darkow, 1967).

An instability index for use in measuring the potential for severe thunderstorms may be obtained by adding (4) and (5):

$$\text{LAPOT} = \frac{2}{p' - p}\left[\frac{\theta_E(p') - \theta_E}{\theta_E(p') + \theta_E} + \frac{\theta_{sE}(p') - \theta_E}{\theta_{sE}(p') + \theta_E}\right] < 0, \qquad (7)$$

This may be called the LAPOT Instability or LAPOT index from its component instabilities: LAtent plus POTential.

The physical significance of the value of the Latent Instability index given by (4) may be illustrated by Fig. 1a which is a plotted tephigram of a sounding in an air mass with latent instability. The dry- and wet-bulb temperature curves are denoted by ABC and DEG. It can be seen that air rising from level H to level K will be colder and denser than the surrounding air. Energy from an outside source must be supplied if it is to reach K; the amount required is represented by the hatched area HJK. The air parcel becomes unstable above level K and rises along the saturated adiabat KML to level L absorbing energy (the energy of instability) supplied by the environment and proportional to the hatched area KBLM; for details see Hewson and Longley (1944). Hence, the value of the Latent Instability index given by (4) for the 700-850 mb layer is proportional to the magnitude of the energy represented by that part of the area KBLM lying between 700 and 850 mb, i.e., area KBM.

The physical significance of the Potential Instability index value obtained from (5) may be illustrated similarly by Fig. 1b. In this case, the layer of air AB is forced to ascend as a whole from the 800-900 to the 600-700 mb levels. It can be seen that, with the dry- and wet-bulb temperature distributions indicated, the layer AB will remain stable until the air at level A reaches saturation at level E. At the same time, the air at level B, having become saturated at level F, ascends along the saturated adiabat from F to level G. The whole layer then becomes absolutely unstable and further ascent takes place spontaneously. The amount of energy supplied by the environment is proportional to the area GEH, so that the value of the Potential Instability index given by (5) for layer AB is then proportional to the energy represented by area GEH.

The value of the LAPOT Instability index given by (7) for a given layer will, therefore, be proportional to the sum of the energies represented by areas such as KBM and GEH.

Each of the above three instability indices may be applied to the 700-850 and 500-700 mb layers as well as to the 500-850 mb layer as a whole. The 850-, 700- and 500-mb levels are those most often used for instability investigations in the atmosphere. The temperature and moisture conditions at the 850-mb level have been found to be representative of the potentially warm, moist low-level air entering the base of the convective storm. Conditions at the 700- and 500-mb levels are representative of the potentially cold, dry air entering the storm downdraft at middle levels and the air near the top of the unstable layer except in the tallest storms. The θ_E, θ_{sE} values at these levels may be readily calculated by computer or from tables (available from the author).

Values of the Latent Instability index in the 700-850 and 500-700 mb layers, calculated from (4), may be used to determine areas of shower and thunderstorm occurrence. Case studies have shown that these usually occur in areas where values for the 700-850 mb layer are less than 10×10^{-5} mb^{-1}. Severe thunderstorms and tornadoes are likely to occur in areas where the Latent and LAPOT Instability indices in the 700-850 mb layer are both less than zero. The probability of severe storm formation increases as the negative values of these indices decrease. The Latent Instability (500-700 mb) index has been found to be a better indicator of showers and thunderstorms in mountainous regions than any of the other θ_E based indices tested: convective storms are likely to occur in such regions when the index values are less than 15×10^{-5} mb^{-1}. The heaviest storms tend to occur remote from mountainous regions (Fujita et al., 1970).

All available surface synoptic and hourly observations were used to determine the weather conditions over central Canada between 1800 GMT 6 August and 0600 GMT 7 August 1969 (Fig. 2). The LAPOT, Latent and Potential Instability (700-850 mb) index values at 0000 GMT 7 August 1969 were obtained for all radiosonde stations in the area. Isopleths of the LAPOT index values (solid lines) were drawn in Fig. 2 mostly within the 10×10^{-5} mb^{-1} isopleth of Latent Instability index values (dashed lines).

It will be seen from Fig. 2 that a strong centre of the LAPOT index values extends from Manitoba across Minnesota and northern Wisconsin to Lake Huron. The centre is located near Sault Ste. Marie about 60 n. mi. northeast of a severe storm outbreak in northern Minnesota between 2200 and 2300 GMT 6 August, with LAPOT and Latent Instability index values ranging from -20 to -50 and 5 to -5, respectively (unit: 10^{-5} mb^{-1}). Showers were found at most observing stations within the "10 isopleth" of Latent Instability index values. Similar relationships between the values of these two indices and convective storm occurrences were found in other case studies.

No relation was found between Potential Instability index values for the 700-850 mb layer and convective storm occurrences. The 10-isopleth is located outside the area of Fig. 2.

3. Statistical Methods and Results

The performance of the Latent, Potential and LAPOT Instability indices ((4), (5) and (7)) have been compared statistically with that of other instability indices whether designed to predict showers, thunderstorms, or severe storms. To do this, ten months of 00 and 12 GMT upper-air (radiosonde) and synoptic data were obtained for 74 North American radiosonde stations situated between 35 and 65°N latitude for two periods: 1 May to 30 September 1969, and 1 May to 30 September 1970. No upper-air or surface synoptic observations were available for 06 and 18 GMT.

The value of an index at a given radiosonde station was assumed to be related to weather occurring there from six hours before to six hours after the time of the relevant sounding (e.g., 06-18 GMT for the 12 GMT sounding), and within a 150-km radius of the station. The absence of the 06 and 18 GMT synoptic observations produced two 6-h gaps in the weather observations, but these were filled in from the cloud and rainfall groups in the 00 and 12 GMT synoptic observations. As an example, for the 00 GMT observation, showers were assumed to have occurred within the 00-06 GMT period if the code figure for R_t was 7 in the rainfall group of the 12 GMT synoptic observation, and if cumulus clouds (C_L = 2 or 8) were reported at 00 GMT; see MANOBS (AES, 1970). (Code R_t = 7 indicates that rain began or ended 6 to 12 h previously.) Thunderstorms would be assumed instead if altocumulus castellanus (C_M = 8) was reported at 12 GMT or cirrus originating from cumulonimbus (C_H = 3) at 00 GMT. Otherwise rain would be assumed. Showers or thunderstorms reported as being in progress at the time of observation, or ending less than six h before (e.g., between 18 and 00 GMT), or cumulonimbus clouds of any amount (C_L = 3 or 9) were taken as an occurrence. A thunderstorm was defined as heavy if accompanied by gusty winds in excess of 33 kt; or rainfall in excess of 38 mm/6 h; or hail in any amount; or tornadoes. Winds of 33 kt were assumed to be gusty. Heavy showers, heavy rain and heavy snow were defined as in MANOBS (AES, 1970).

Two types of index were not included in the test - those requiring graphs or diagrams to evaluate them, viz.: the Similä, Millar and Starrett, and Hanssen indices (see Saunders, 1966), and those requiring forecast values, viz.: the Sly Index (Sly, 1964). The Lifted Index (Galway, 1956) was included even though requiring the forecast maximum temperature, assuming that the maximum surface temperature lies on the dry adiabat through the observed temperature at 850 mb.

The following indices and parameters were also included in the test: the Boyden (Boyden, 1963), Delta (Jenrette, 1960), George (George, 1960), Jefferson (Jefferson, 1963a), Modified Jefferson (Jefferson, 1963b), Showalter (Showalter, 1953), SWEAT (Bidner, 1970), Total totals (Miller, 1967) and the Latent, Potential and LAPOT Instability indices; the moisture deficits and the material differential advections (Fujita et al., 1970) in the 700-850, 500-700, 400-500 and 500-850 mb layers; the total energy in the 500-850 mb layer; the vertical velocity at 600 mb, ω_6 (Harley, 1965); the depth of the moist layer above the lifting condensation level; and the depth of the unstable layer. Various combinations of indices were also tested.

The moisture deficit in a layer was taken as the difference between the change in saturation mixing ratio r_w along the psuedo-adiabat through the wet-bulb temperature at the base of the layer $(\Delta r_w)_{\theta_w}$, and the change in mixing ratio r between the base and top of the layer (Δr), viz., $(\Delta r_w)_{\theta_E} - (\Delta r)$, where θ_E is the saturated potential temperature of the pseudo-adiabat. The depth of the unstable layer was taken as the difference between the pressures at the lifted condensation level PCL, and the level where T(ascent curve) - T(environment) is a maximum, $P(\Delta T)_{max}$, i.e., PCL - $P(\Delta T)_{max}$. The depth of the moist layer was taken as PCL - P(J), where P(J) is either 400 mb or the highest level in the sounding at which a dew point is reported. It was assumed that measurable moisture may extend to 400 mb, the maximum level of the available data. These moisture parameters were included because of their direct relationship to storm intensity and rain occurrence. All values were obtained by computer.

The total number of observations of each weather type and the total number of observations for all radiosonde stations are given in Table 1. Each entry in the table is a range of numbers, since incomplete temperature and moisture observations from the atmospheric sounding associated with a specific weather type may permit computation of one index but not another. The total number of observations N of each index is the sum of the number of observations accompanied by weather occurrences and those that are not. The value of N will vary from index to index.

Table 1

Range of total number of observations of each index accompanied by weather occurrences over all indices. N for each index is the total number of observations accompanied by weather occurrences plus non-occurrences (see text).

Weather Type	Range of Index
Showers or thunderstorms	11,311 - 9,745
Heavy showers or thunderstorms	3,743 - 3,428
Rain or snow	104 - 89
Heavy rain or snow	11 - 10
Range of N	33,879 - 30,220

The biserial correlation coefficient was used as a measure of the connection between the value of an instability index (the observation), and the occurrence of showers or thunderstorms (or rain, snow, severe thunderstorms, etc.). This statistic is given by the following relation (Brooks and Carruthers, 1953):

$$r_b = \frac{(M_1 - M_0)}{\sigma_b \cdot z_b} p \cdot q, \qquad (8)$$

where M_i is the mean of the index values corresponding to showers or thunderstorms; M_0 is the mean of those not accompanied by showers or thunderstorms;

p is the proportion of index values accompanied by convective storms; q is the proportion not so accompanied; σ_b is the standard deviation of all index values; and z_b is the height of the ordinate of a normal frequency curve, with standard deviation unity, which cuts off proportions p and q of the observations. The occurrence and non-occurrence of showers or thunderstorms are assumed to represent two broad classes of a series of observations that are normally distributed; see Brooks and Carruthers (1953).

The biserial correlation coefficient between index value and convective storm occurrence was used to determine which instability indices correlate most highly with convective storms. The skill score was also used as an additional test of the relation between these two parameters because of its convenience and once widespread use. The same set of observations was used to obtain both scores. The skill score S of a specific index is given by Brier and Allen (1951):

$$S = \frac{CF - CC}{N - CC}, \quad CC = \frac{x \cdot p}{N} + \frac{y \cdot q}{N}, \tag{9}$$

where CC is the number of forecasts correct by chance, CF is the number of correct forecasts, x is the number of observed occurrences, y is the number of observed non-occurrences, p is the number of forecast occurrences, q is the number of forecast non-occurrences, and N is the total number of observations.

The question arose as to the method of computing the skill score when the observations are assumed to be forecasts. This may be done by first determining a critical value of the index, considered as that index value accompanied by showers or thunderstorms 50 per cent of the time. This value was obtained from a line graph of percentage frequency of shower and thunderstorm occurrence plotted against index value; see Figs. 3, 4 and 5. Observations of showers or thunderstorms for index values on the unstable side of the critical value are considered to be correct forecasts; those on the stable side as incorrect. Similarly, when no showers or thunderstorms are observed for values on the stable side of the critical value, these are considered correct forecasts; when these are on the unstable side the forecasts are incorrect. The forecasts are then judged against chance (Brier and Allen, 1951).

The frequency distribution tables produced by the computer were used to obtain polynomial regression equations of best fit corresponding to the frequency distributions for the indices. These equations are required in order to develop a probability method of shower and thunderstorm prediction.

A set of computer programs was written to prepare the data, perform the necessary computations and print out the results. The following procedures were carried out:

a) Quality control of the radiosonde data by vertical and horizontal consistency checks.

b) Computation of the value of each index for each radiosonde station and each observation time. The maximum and minimum values of each index were also determined.

c) Determination of the weather category at each radiosonde station and observation time.

d) Production of a frequency distribution table and a diagram of shower and thunderstorm frequency versus index value for each index.

A computer program performs the following operations:

i. Scales the range of values of each index into 100 equal groups and obtains the total number of values and the mean value of each index in each group.

ii. Determines the number of weather observations in each group and the total of all groups.

iii. Computes the percentage occurrence of showers and thunderstorms in each group.

iv. Prints a table of shower and thunderstorm frequency for each of the 100 groups and plots percentage frequency distributions of shower and thunderstorm occurrence versus index value.

Line graphs were drawn for some of the frequency distributions; e.g., Figs. 3, 4 and 5.

e) Calculation of the biserial correlation coefficient r_b, and the skill score S (where possible) and the critical value CV for each index. Skill scores could not be computed for indices with r_b scores less than ~ 0.2, e.g., the vertical velocity ω_6 with an r_b score of 0.196. No critical value at the 50% level could be obtained from such indices.

f) Determination of the polynomial regression equations of best fit for frequency distributions of percentage shower and thunderstorm occurrence versus index value for instability indices selected from Table 2. A computer carried out the following operations:

i. Evaluation of the least squares polynomials of degree less than four using orthogonal polynomials.

ii. Printout of the observed Y and predicted values YP of shower and thunderstorm frequencies for each index value and the sub-group as well as the value of the deviations Y - YP. The polynomial with the lowest values of the deviations Y - YP is taken as the polynomial of best fit. A weighting factor from 0 to 4 was assigned to each data point of the distribution according to its scatter or vertical distance from a mean curve determined by inspection. The value zero was assigned to points on the mean curve.

Results for the twelve indices with the highest correlation coefficients based on the 74-station data set are given in Table 2, along with corresponding results for a 65-station data set obtained by omitting stations from seven southwestern States, viz., Arizona, California, Colorado, Nevada, New Mexico, Texas and Utah.

Table 2

Biserial correlation coefficient r_b, skill score S and critical value CV for selected instability indices for two data sets based on different frequency distributions of showers and air-mass thunderstorms versus index value.

Index	74-station set			65-station set		
	r_b	S	CV	r_b	S	CV
Modified Jefferson (M)	0.642	0.246	31.5	0.591	0.238	28.0
Latent (500-700 mb)	-0.618	0.286	-3.5	-0.514	0.235	8.0
George (K)	0.594	0.257	36.0	0.573	0.218	28.0
LAPOT (500-700 mb)	-0.587	0.253	-5.0	-0.524	0.220	9.0
Total totals (TT)	0.538	0.255	54.0	0.520	0.192	49.0
Jefferson	0.526	0.211	36.0			
Potential (500-700 mb)	-0.512	0.160	-17.0	-0.448	0.141	-1.5
Lifted	-0.503	0.108	-7.5	-0.504	0.239	0.0
Showalter	-0.459	0.107	-7.0			
SWEAT	0.453	0.077	330.0	0.448	0.118	257.0
Latent (700-850 mb)	-0.424	0.222	-18.0	-0.326	0.103	-16.0
LAPOT (500-850 mb)	-0.422	0.110	21.0			

It can be seen from Table 2 that the r_b for the Jefferson Index increases from 0.526 to 0.642 on adding the term $(T - T_d)_7/2$ to account for the moisture at the 700-mb level (Jefferson, 1963b). Similarly, modifying the Showalter Index by substituting the mean dew point in the lowest 100-mb layer for the 850-mb dew point decreases r_b from -0.459 to -0.503 for the Lifted Index. The scores for the LAPOT (500-850 mb) index have been omitted from the 65-station set, since no critical value could be obtained. It can also be seen from Table 2 that, except for the Lifted Index, a general increase in r_b and S occurs if data from nine radiosonde stations in the southwestern United States are included. This suggests that most of the indices in Table 2 measure instability more efficiently in that part of the United States.

The M and K indices were designed for air-mass thunderstorms and to provide high values when deep moist layers are present; see Jefferson (1963b) and George (1960). It will be seen that these two are among the four top-scoring indices in Table 2, the other two being the Latent and LAPOT indices for the 500-700 mb layer. The SWEAT index, formed by adding wind speed and wind shear terms to the TT index does not score as well as the Lifted Index with respect to showers and air-mass thunderstorms, because it was designed for severe thunderstorms and tornadoes where wind speed and shear are more significant (Bidner, 1970). The Potential Instability (700-850 mb) index was found to be uncorrelated with convective storms: $r_b = 0.009$.

The standard error σ_r of the biserial correlation coefficients given in Table 2 were computed from Equation (168) given by Brooks and Carruthers (1953). For the highest coefficient ($r_b = 0.642$), the standard error is 0.0156; for the lowest ($r_b = -0.326$), it is 0.0120; therefore all coefficients are highly significant.

The computer procedures were repeated to obtain corresponding biserial correlation coefficients for severe storms (see Table 3).

Table 3

Biserial correlation coefficients r_b for eight highest scoring instability indices: for severe thunderstorms, 74-station data set.

Index	r_b	Index	r_b
SWEAT	0.339	ω_6	-0.242
Latent (500-700 mb)	-0.319	LAPOT (400-500 mb)	-0.241
Latent (400-500 mb)	-0.290	Boyden	0.234
Modified Jefferson (M)	0.260	George (K)	0.230

No skill scores could be calculated for the severe storm occurrences, since no critical values could be obtained for any of the indices; the only thunderstorm frequency distribution rising above the 25% occurrence value (along the ordinate) was that of the SWEAT index. Not even this distribution reached the 50% value (cf. Fig. 4). However, the standard errors of the coefficients given in Table 3 (from $\sigma_r = 0.0274$ for $r_b = 0.339$ to $\sigma_r = 0.0285$ for $r_b = 0.230$) show that these coefficients also are highly significant.

It will be seen from Table 3 that the SWEAT index is the best predictor of heavy or severe thunderstorms, in agreement with the results of David and Smith (1971) who evaluated different instability predictors of severe storms. The relatively high scores for the Latent and LAPOT Instability indices in the 400-500 mb layer reflects the fact that instability extends to higher levels in severe storms than in air-mass thunderstorms. The r_b scores for the M and K indices are slightly lower than that of the SWEAT index (cf. Table 2), a result indicating the increased significance for severe storm formation of the wind speed and wind shear terms in the SWEAT index over the simple moisture terms in the M and K indices. The parameter ω_6 is a measure of the large-scale vertical velocity, one of the critical conditions for severe storm formation; see Newton (1967). The relative effectiveness of the Boyden Index in the prediction of severe storms may be traced to the significance of the mean lapse rate below the 700-mb level in such cases (Boyden, 1963).

The number of rain or snow occurrences found were insufficient to produce significant statistics; see Table 1. However, examination of the frequency distribution table for rain or snow occurrence versus index value (not shown) suggested that the depth of the moist layer above the lifting condensation level was the best indicator of rain or snow occurrence ($r_b = 0.186$, $\sigma_r = 0.096$).

The polynomial regression curves of best fit for the Modified Jefferson, Latent Instability (500-700 mb) and Latent Instability (700-850 mb)

indices are illustrated in Figs. 3, 4 and 5, respectively. These figures show that the polynomial curves fit the frequency distributions reasonably well only within specific limits that are indicated by ticks on the curves. Outside these limits the distributions become very irregular and the convective storm occurrences few in number. A constant percentage of shower and thunderstorm occurrences is assumed for index values outside these limits. For example, a convective storm occurrence of 90% has been assumed for an M index value above 42 (Fig. 3).

To convert from an index value I to a specific subrange along the abscissa SR, and vice versa, the following equation may be used:

$$SR = 100 \cdot \frac{(I - M_N)}{M_X - M_N}, \qquad (10)$$

where M_X and M_N are the maximum and minimum values of the index in the subrange, respectively. M_X and M_N are given in Table 4 for selected indices:

Table 4

Maximum and minimum values of selected instability indices.

Index	Maximum	Minimum	Unit
Modified Jefferson	121.0	-24.0	°C
Latent Instability (500-700 mb)	45.0	-14.0	10^{-5} mb^{-1}
George	50.0	-50.5	°C
SWEAT	751.0	0.0	
Latent Instability (700-850 mb)	160.0	-75.0	10^{-5} mb^{-1}

For example, using Table 4 and (10), the subrange corresponding to the Modified Jefferson index value of 42 is found to be 45.5 (cf. Fig. 3).

The polynomial regression equations for instability indices selected from Table 2 and their corresponding ranges of application are given in Table 5, together with the assumed values of percentage storm occurrence outside these ranges.

The two regression polynomials for the Latent Instability (700-850 mb) index listed in Table 5 correspond to the two branches of the U-shaped curve of Fig. 5. The junction between the two curves is indicated by the tick at index value 29 (subrange 44).

No values of percentage storm occurrence are given in Table 5 for negative values of the SWEAT Index, since this cannot have a value less than zero; see Bidner (1970). The results illustrated in Figs. 3-5 and Table 5 are based on data from the 65-radiosonde-station set.

Table 5

Regression polynomial of best fit for instability indices selected from Table 2 and their corresponding ranges of application. Assumed values of percentage occurrence of showers and thunderstorms are given for index values outside these ranges.

Index	Range of Application	Regression Polynomials and Assumed Values of Percentage Storm Occurrence
Modified Jefferson	< 4.0	10%
	4.0 to 42.0	$12.644 - 0.5936X + 0.01723X^2 + 0.00338X^3 - 0.0000568X^4$
	> 42.0	90%
Latent (500-700 mb)	< -10.0	80%
	-10.0 to 40.0	$65.318 - 1.9601X - 0.048687X^2 + 0.0025709X^3 + 0.000024618X^4$
	> 40.0	15%
George	< -1.5	10%
	- 1.5 to 41.0	$17.935 + 0.42067X - 0.015494X^2 + 0.001771X^3 - 0.00002347X^4$
	> 41.0	90%
SWEAT	0.0 to 490.0	$12.97 + 0.138X$
	> 490.0	80%
Latent (700-850 mb)	< -43.0	75%
	-43.0 to 29.0	$36.733 - 0.83353X + 0.001688X^2 + 0.00017327X^3$
	29.01 to 79.0	$69.499 - 3.8707X + 0.10575X^2 - 0.001109X^3 + 0.0000040188X^4$
	> 79.0	20%

The frequency distribution for the Latent Instability (700-850 mb) index based on the full data set showed two well-marked frequency maxima, with the one at the higher (stable) index value being the greater. The high percentage of convective storm occurrences associated with the secondary maximum was assumed to be due to high-level instability above 700 mb. To test the validity of this assumption, the computer procedure was re-run omitting data from nine radiosonde stations in the southwestern United States, a region with a high incidence of warm, dry air masses and high-level showers and thunderstorms in summer. Only 657 of the original 2,896 occurrences remained in the secondary maximum situated above index value 29, a reduction of 77%; see Fig. 5. Convective storms associated with this reduced maximum are assumed to be either showers and thunderstorms due to instability above 700 mb or storms initiated by frontal action. No appreciable change occurred in the rest of the distribution. Much smaller secondary frequency maxima were found in the distributions for the Modified Jefferson and Latent Instability (500-700 mb) indices. The number of observations involved in each case were reduced about 10% by subtracting the nine-station data, but the form of the two secondary maxima remained relatively unchanged; see Figs. 3 and 4.

The polynomial for the George index given in Table 5 is based on the 74-station data set. Omission of data from the nine stations in the southwestern United States does not appreciably alter the form of the resultant frequency distribution or the corresponding regression polynomial for this index.

An observed index value can be converted to a percentage probability of shower and thunderstorm formation by substituting it in the appropriate polynomial of Table 5. For example, the probability of shower or thunderstorm occurrence corresponding to a value of 2 for the Latent Instability (700-850 mb) index is 35%. Probabilities obtained in this way from values of the Modified Jefferson or George indices may be used as predictors of air-mass convective storms using observed weather data for North America.

6. Summary of Results

Instability indices based on the equivalent potential temperature were obtained, and then compared statistically with many commonly used indices by utilizing ten months of upper-air and synoptic data from 74 North American radiosonde stations situated between 35 and 65°N.

Polynomial regression equations were fitted to the frequency distributions relating index values and percentage shower and thunderstorm occurrence for four of the highest scoring indices. The equations can be used to obtain the probabilities of shower and thunderstorm occurrence from observed values of each index.

The LAPOT Instability index was developed to test its value as an indicator of instability, but the Latent Instability index, one of its components, was found to be superior; cf. Tables. 2 and 3.

Values of all parameters used in this study can easily be obtained by computer. Tables for the ready computation of the equivalent potential temperature from values of the temperature and dew point at the 850-, 700- and 500-mb levels can be obtained from the author on request.

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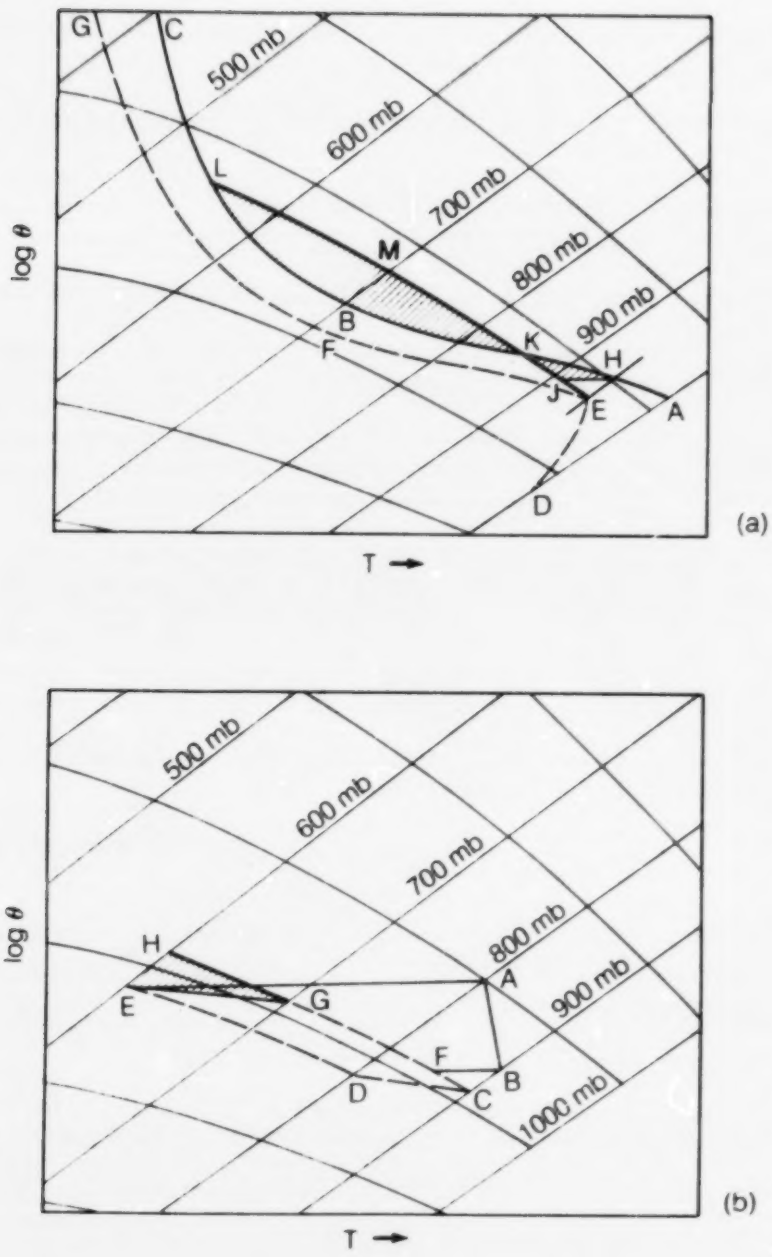

Fig. 1 Tephigrams of atmospheric temperature and moisture distributions for: (a) Latent Instability; (b) Potential Instability.

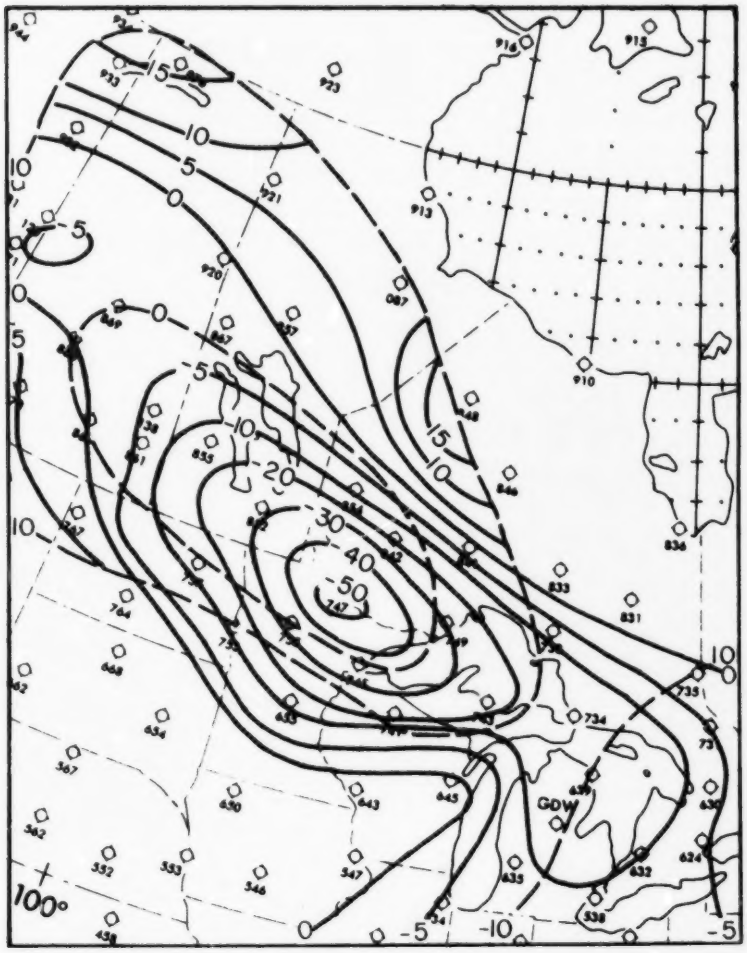

Fig. 2 LAPOT instability index for the 700-850 mb layer for 0000 GMT 7 August 1969 (solid isopleths at 10 x 10^{-5} mb^{-1} intervals). The dashed lines are the 10 x 10^{-5} mb^{-1} and zero isopleths of the Latent Instability index for the 700-850 mb layer.

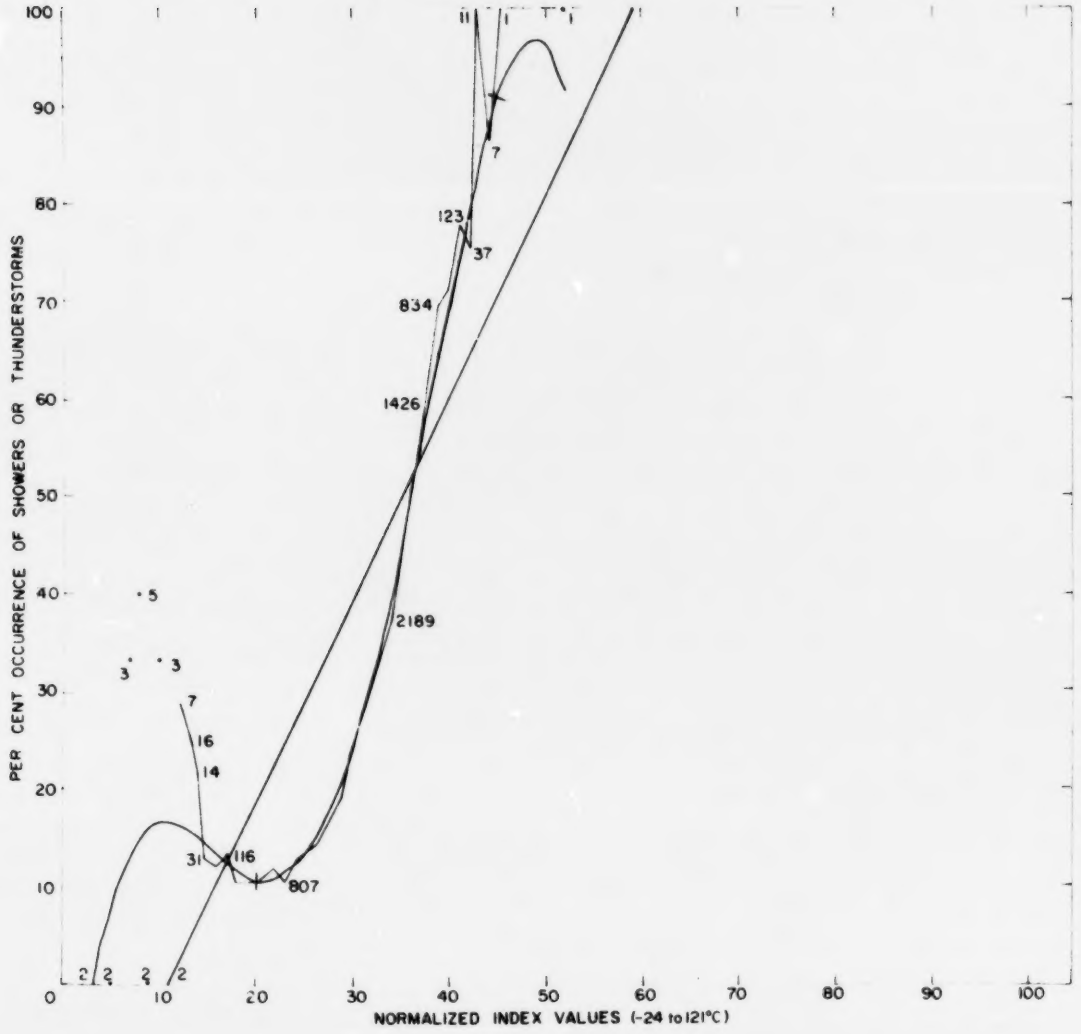

Fig. 3 The frequency distribution of percentage occurrence of showers and thunderstorms for values of the Modified Jefferson index in the range -24 to 121°C. This range has been divided into 100 subranges. The frequencies of showers and thunderstorms are marked for selected subranges. The polynomial regression curve of best fit is drawn through the data points of the distribution (see text).

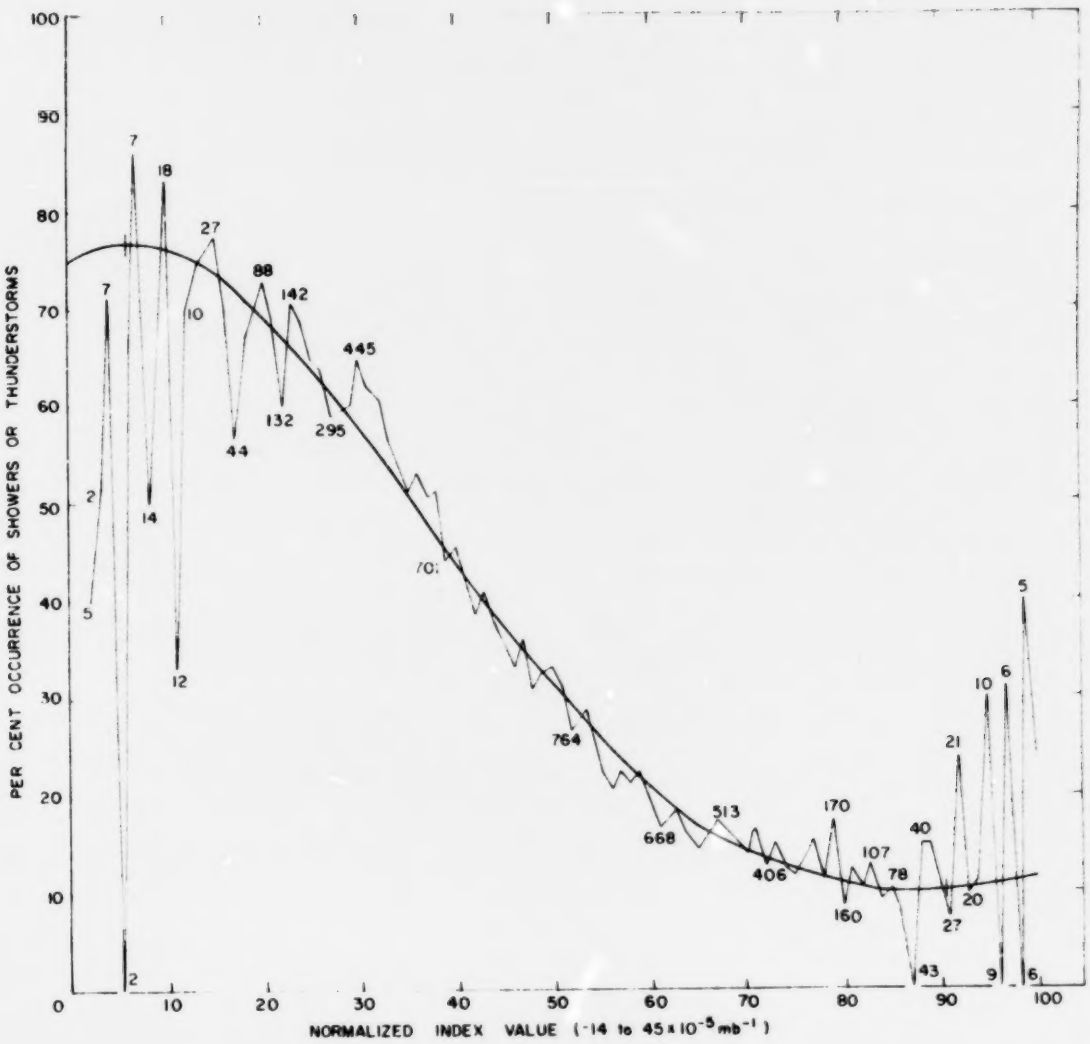

Fig. 4 As for Fig. 3, but for the Latent Instability index for the 500-700 mb layer. The range of index values is from -14 to 45 x 10^{-5} mb^{-1}.

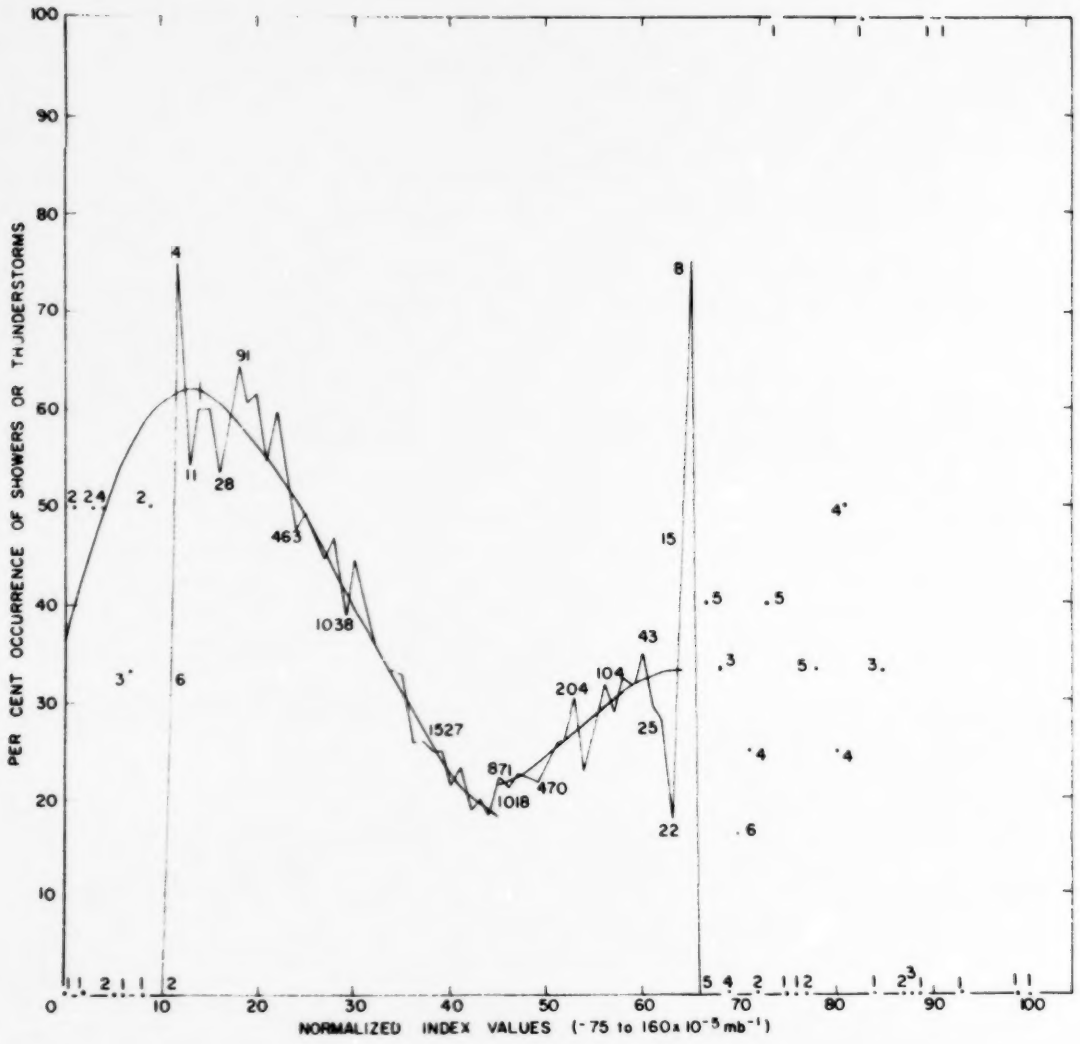

Fig. 5 As for Fig. 3, but for the Latent Instability index for the 700-850 mb layer. The range of index values is -75 to 160 x 10^{-5} mb^{-1}. Two regression curves are required to fit the U-shaped distribution (see text).

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